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Generalizing the Golden Ratio

Abstract: Given a number $a + b\text{sqrt}(q)$ $(a, b \in \mathbf{Q},\ \text{sqrt}(q) \notin \mathbf{Q})$, does there exist a sequence of real numbers convergent to $a + b\text{sqrt}(q)$? The Golden Ratio, also known as $\phi = (1 + \text{sqrt}(5))/2$, is one of mathematics' most famous constants. While this number possesses many beautiful properties, we shall be focusing on its connection to the Fibonacci sequence. The Fibonacci sequence, f_n, is defined as

$$f_0 = 0,\ f_1 = 1,\ \text{and}\ f_n = f_{n-2} + f_{n-1}\ \text{for}\ n \geq 2.$$

It is well known that the limit as $n \to \infty$ of f_n/f_{n-1} equals ϕ, and this ultimately provides the motivation for our question. Given an element of a simple finite extension of $\mathbf{Q}$, can we define a sequence S_n such that the limit as $n \to \infty$ of S_n/S_{n-1} equals $a + b\text{sqrt}(q)$?